MEM6804 Modeling and Simulation for Logistics and Supply Chain: Theory & Analysis

Sino-US Global Logistics Institute
Shanghai Jiao Tong University

Spring 2021 (full-time)

Assignment 4

Due Date: June 2 (in class)

Instruction

(a) You can answer in English or Chinese or both.
(b) Show enough intermediate steps.
(c) Write your answers **INDEPENDENTLY**.

. . . .

Question 1 (20 points)

Suppose $X_1, \ldots, X_n$ are an iid sample from $\mathcal{N}(\mu, \sigma^2)$. Find the estimators of μ and σ^2 using MLE. (*Write down the rigorous derivation steps.*)

Question 2 (15 points)

Suppose $X_1, \ldots, X_n$ are an iid sample from Weibull(α, β) in shape & scale parametrization. The density function of Weibull(α, β) is $f(x) = \alpha\beta^{-\alpha}x^{\alpha-1}e^{-(x/\beta)^\alpha}$, $x > 0, \alpha > 0, \beta > 0$. Suppose the parameter α is known. Find the estimator of β using MLE. (*Write down the rigorous derivation steps; You will notice that if α is unknown and needs to be estimated together using MLE, then it requires some numerical method like Newton's method.*)

Question 3 ($10 + 15 = 25$ points)

For the illustrative example on lecture note Lec5, the first considered exponential distribution is rejected. We then consider the Weibull family. The density function of Weibull(α, β) in shape & scale parametrization is $f(x) = \alpha\beta^{-\alpha}x^{\alpha-1}e^{-(x/\beta)^\alpha}$, $x > 0, \alpha > 0, \beta > 0$. Suppose the parameters are estimated from the data via MLE: $\hat{\alpha} = 0.525$, $\hat{\beta} = 6.227$.

(1) Make the Q-Q plot. (*Note*: Show the necessary calculation. Use Excel or other software/language to draw the final plot.)

(2) Use K-S test to see if we would like to reject Weibull$(0.525, 6.227)$ at level of significance $\alpha = 0.1,\ 0.05,\ 0.01$. (*Note*: You can use Excel or other software/language to compute the value of test statistic D; but implement the formula of D by yourself. The $(1-\alpha)$-quantile of D is $d_{n,1-\alpha} = c/\sqrt{n}$, and the value of c is given in the following table.)

	$1-\alpha$			
n	**0.900**	**0.950**	**0.975**	**0.990**
10	0.679	0.730	0.774	0.823
20	0.698	0.755	0.800	0.854
50	0.708	0.770	0.817	0.873
∞	0.715	0.780	0.827	0.886

Question 4 (10 points)

Let $F_n(x)$ be the empirical CDF (check the formal definition in the updated Lec 5, page 13/57). If $F(x)$ is the true underlying distribution. Prove that, for all $x \in \mathbb{R}$, $F_n(x) \xrightarrow{a.s.} F(x)$. (*To understand this question, you need to note that, if you stand at time 0, then for any given x, $F_n(x)$ is a random variable.*)

Question 5 ($3 + 10 + 2 = 15$ points)

Suppose we run a steady-state simulation, and observe discrete outputs $Y_1, Y_2, \ldots, Y_n$ in one simulation run. Suppose the initialization bias can be ignored. We use $\bar{Y} = \frac{1}{n}\sum_{i=1}^{n} Y_i$ as the point estimator of the steady-state performance measure ϕ. The following is a mistake that one will easily make: Calculate $S^2 = \frac{1}{n-1}\sum_{i=1}^{n}(Y_i - \bar{Y})^2$, and use $\bar{Y} \pm t_{n-1,1-\alpha/2}\frac{S}{\sqrt{n}}$ as the $1-\alpha$ confidence interval for ϕ.

(1) Why is that a mistake? Briefly explain why.

(2) Suppose $Y_1, Y_2, \ldots, Y_n$ are identically distributed and positively correlated, prove the following: $\mathbb{E}[S^2] < \mathrm{Var}(Y_1)$, $\mathbb{E}[S^2/n] < \mathrm{Var}(\bar{Y})$.

(3) For the situation in (2), if we use $\bar{Y} \pm t_{n-1,1-\alpha/2}\frac{S}{\sqrt{n}}$ as the confidence interval, what is the consequence?